

(NASA-CR-197609) COMPARISON OF
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(Kansas Univ.) 11 p

N95-19363

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Comparison of Dynamical Approximation Schemes for Non-Linear Gravitational Clustering

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ABSTRACT

We have recently conducted a controlled comparison of a number of approximations for gravitational clustering against the same n -body simulations. These include ordinary linear perturbation theory (Eulerian), the adhesion approximation, the frozen-flow approximation, the Zel'dovich approximation (describable as first-order Lagrangian perturbation theory), and its second-order generalization. In the last two cases we also created new versions of the approximation by truncation, i.e., smoothing the initial conditions by various smoothing window shapes and varying their sizes.

The primary tool for comparing simulations to approximation schemes was crosscorrelation of the evolved mass density fields, testing the extent to which mass was moved to the right place. The Zel'dovich approximation, with initial convolution with a Gaussian e^{-k^2/k_G^2} , where k_G is adjusted to be just into the nonlinear regime of the evolved model (details in text) worked extremely well. Its second-order generalization worked slightly better.

All other schemes, including those proposed as generalizations of the Zel'dovich approximation created by adding forces, were in fact generally worse by this measure. By explicitly checking, we verified that the success of our best-choice was a result of the best treatment of the phases of nonlinear Fourier components. Of all schemes tested, the adhesion approximation produced the most accurate nonlinear power spectrum and density distribution, but its phase errors suggest mass condensations were moved to slightly the wrong location. Due to its better reproduction of the mass density distribution function and power spectrum, it might be preferred for some uses.

We recommend either n -body simulations or our modified versions of the Zel'dovich approximation, depending upon the purpose. The theoretical implication is that pancaking is implicit in all cosmological gravitational clustering, at least from Gaussian initial conditions, even when subcondensations are present.

Key words: cosmology:theory-dark matter-galaxies:clustering-large scale structure of the universe.

I. INTRODUCTION

The gravitational instability picture has emerged as the dominant paradigm for understanding the growth of structure in the universe from the small-amplitude fluctuations present at recombination. When the density fluctuations are very small, linear perturbation theory (Eulerian) works well (for a summary see Peebles 1980, 1993). In the deeply non-linear regime, n -body simulations are usually used, perhaps with simulated hydrodynamics added. Simulations in general can suffer from a typical fault of numerical results that they can be quite correct without our understanding why and therefore be difficult to generalize. Even worse, without approximate analytic solutions as a check, errors may be unrecognized.

Analytic or quasi-analytic nonlinear approximations occupy an intermediate position. They can capture some non-linear effects correctly in a way that permits us to understand and generalize from them more easily. They can also be used to provide boundary conditions for simulations, start them at a more advanced state, or generate large numbers of approximate realizations, for statistical purposes. It is for this reason that they are worth proposing; it is also for this reason that they are worth testing objectively in a way that allows comparisons. In this *Letter* we report succinctly the main results of such a project.

In Coles, Melott and Shandarin (1993), hereafter CMS, we began by studying the usual linear (Eulerian) perturbation theory, hereafter L, the lognormal (Coles and Jones, 1991) approximation (which was basically an exponentiation of L) and the Zel'dovich (1970) approximation. The lognormal approximation was particularly poor and will not be considered further in this *Letter*.

The Zel'dovich approximation (hereafter 1ZA) did surprisingly well, especially in a modified form in which the initial conditions were smoothed at a scale close to the threshold of nonlinearity. A number of other schemes have been suggested recently which are designed to be improvements on 1ZA.

II. APPROXIMATIONS STUDIED

Brief verbal descriptions will be presented for the many approximations tested. For full details please see the citations.

Linear theory (L) results from perturbing the equations of motion. The result is that (for $\Omega = 1$, but easily generalized) the fluctuations in merely grow in amplitude $\delta \propto a(t)$ where a is the scale factor. A velocity is of course also implied, but does not produce the associated δ except to first order. See Peebles (1980, 1993).

The Zel'dovich (1970) approximation (1ZA) consists of the assumption that the velocity taken from linear theory continues. In comoving coordinates \bar{x} , $\Omega = 1$, this reduces to $d\bar{x}/a = \text{constant}$. The density is then derived from the position of mass elements. The assumption that the velocity will behave in this way seem especially appropriate when the acceleration field is constant over large regions, i.e. it was associated with universes in which there is extensive damping of small-scale density fluctuations, such as those with adiabatic baryon perturbations or hot dark matter. 1ZA was tested (up to resolution limits) by CMS.

Buchert (1992) has provided the derivation that Zel'dovich never presented for his approximation as resulting from first-order perturbation theory around the Lagrangian equations of motion. Furthermore, he has extended them to second order which includes tidal forces and even to third order. We have studied 2ZA and 3ZA as well for damped models. The 2ZA approach is being studied now for the power-law spectra reported here (Melott, Buchert, and Weiss, 1994).

CMS introduced the “truncated Zel'dovich approximation” (hereafter 1TZA) by smoothing the initial conditions on the scale of nonlinearity before applying the approximation. This consists of destroying information about deeply nonlinear modes the approximation cannot handle. It was inspired by previous observations that the pattern of arrangement of clumps in hierarchical clustering simulations resembled those of the pancakes in simulations with the same phases but all the initial power set to zero for modes that had gone nonlinear (Melott and Shandarin 1990; Beacom *et al.* 1991; Little, Weinberg and Park 1991; see also Melott and Shandarin 1993). They found that 1TZA worked extremely well, outperforming everything else. More recently Melott, Pellman and Shandarin (1994, hereafter MPS) searched for the optimum scale and shape of truncation, resulting in considerable improvement over the CMS formulation by using a Gaussian smoothing of the initial conditions. We shall refer to this optimized form as 1TZA. It is important to stress that the results of 1TZA do not resemble those of 1ZA for most spectra. It is for this reason that claims that an approximation is better than the Zel'dovich approximation are not very meaningful. 1ZA and 2ZA are quite bad for power-law spectra $n > -1$ (see part III).

The “frozen flow” approximation (FFA) Matarresse *et al.* (1992) is one in which the particle follow streamlines of the original linear velocity field. This takes multiple steps, because the particles' velocity depends on their position. It is not strictly an analytic approximation but it is so fast we will include it. FFA was tested by Lucchin *et al.* (1994) hereafter LMMM.

The adhesion approximation (AA) Gurbatov, Saichev, and Shandarin 1989) contains ZA as a core, but adds an effective viscosity term which causes intersecting flows to “stick.” This is an attempt to correct the first serious fault of ZA, that particles continue past shell crossing in the approximation, but are slowed by gravity and fall back in the real world. Melott, Shandarin, and Weinberg (1994) tested the adhesion approximation using the method of Weinberg and Gunn (1990).

Although plans exist to study it in the same way, we have not delayed this letter to include results for the “frozen potential” FP method (Brainerd *et al.* 1994, Bagla and Padmanabhan 1994). Although it may perform well, it is very far from an analytic approximation. It really consists of doing an n -body simulation without re-solving for the potential at each step. This takes advantage of the fact that the potential is constant to linear order, and dominated by longwave modes which are little affected by nonlinearities. But analytic solutions do not exist, and solving for the potential is easy with modern numerical methods. It might provide insight, but it will never replace an analytic approximation *or* an n -body simulation.

One might propose generalizations of FFA, AA, or FP in which the initial potential

is smoothed. But in this case they would revert to something very close to 1TZA. The whole purpose for creating them was to handle the nonlinear modes, which 1TZA simply removes from the initial conditions.

When trying to restore initial conditions from the evolved state, conclusions presented herein do not apply. See Melott (1993).

III. PROCEDURES

All of our approximations are compared to a group of n -body simulations more fully described in Melott and Shandarin (1993). These were 128^3 particle runs with Gaussian initial conditions characterized by power-law spectra of density fluctuations (see Peebles 1980) $P(k) \propto k^n$ for $n = -2, -1, 0, +1$ which includes most cases of cosmological interest. Conclusions about likely behavior under specific physical scenarios can be reconstructed from the power-law slopes just going nonlinear at the moment under consideration. The n -body simulation and the approximations were compared primarily by cross-correlation

$$S = \frac{\langle \delta_1 \delta_2 \rangle}{\sigma_1 \sigma_2} , \quad (1)$$

where $\delta_i = \langle \delta_i^2 \rangle^{1/2}$ and δ_i is the pixellated density of the simulation or approximation. If they are identical, $S = +1$. We allowed for the fact that condensations might be just slightly in the wrong place by calculating S for both fields with a wide variety of smoothing lengths.

Some statistical analysis was also done, including the power spectrum and density distribution function. These will not be shown here, but can be found in the various more detailed studies.

The approach used here has a number of advantages over those used for testing in most of the approximation proposals. Most obviously, they are all tested against the same initial conditions with the same methods, so they can be compared with one another. We have also checked for detailed dynamical agreement rather than just a similar visual appearance or power spectrum. One of the things we learned was that power spectra can be similar for two approximations while *phases* can be in much better agreement in one scheme than in another. Crosscorrelation is sensitive to phase information.

IV. RESULTS

In Figure 1 we show the crosscorrelation S as a function of σ_1 , the *rms* density fluctuations of the smoothed n body for two indices $n = -1$ and $+1$. The case $+1$ is the most demanding and shows the differences. The case -1 is of interest because it is probably close to the slope just going nonlinear today.

It is clear that TZA is the best choice by this criterion. 1TZA, or just TZA as described by MPS, consists of Gaussian smoothing near the scale of nonlinearity. We define k_{nl} by

$$a^2(t) \int_0^{k_{nl}} P(k) d^3 k \equiv 1 \quad (2)$$

where P is the power in the initial conditions. Therefore $k_{n\ell}$ is the wavenumber where $\sigma = 1$ by extrapolation using linear theory. MPS found that the optimum smoothing was convolution of initial density by a Gaussian $e^{-k^2/2k_G^2}$ with $k_G = 1.5 k_{n\ell}(n = -2, -1)$, $1.25k_{n\ell}(n = 0)$ or $k_{n\ell}(n = +1)$. As the maximum is fairly broad, one could use $k_G = 1.25k_{n\ell}$ for all cases without serious error. In the case of non-power law spectra we recommend examining the local slope at $k_{n\ell}$.

This approximation scheme is within reach of anyone who has code to implement the Zel'dovich approximation, and a Fast Fourier Transform. It is extremely simple to implement, and takes about as much CPU time as one step in an n -body simulation. MPS also checked for agreement of particle positions and velocities, and found generally small errors. Borgani et al (1994) have already used 1TZA to generate large ensembles to test cluster-cluster correlations, and found good agreement with the large n -body simulations of Klypin and Rhee (1993).

Preliminary results indicate that 2TZA (Second order Lagrangian perturbation theory with Gaussian smoothing of initial conditions) is a small but measurable improvement over 1TZA (Melott, Buchert, and Weiss 1994). It is important to also note that as reported in detail by Melott, Shandarin, and Weinberg (1994), the adhesion approximation more accurately reproduces the power spectrum and mass density distribution function of the simulations than does 1TZA. For spectral index $n < -1$, the crosscorrelation is not bad, and for this reason the adhesion approximation might be preferred for certain purposes. For most purposes, use of 1TZA is a simply implemented major improvement over approximations now in use.

V. ACKNOWLEDGEMENTS

This research was supported in the USA by NASA (NAGW-2923), NSF (AST-9021414). For my collaborators it was also supported by (in the US) the W.M. Keck Foundation and NSF grant PHYS 92-45317. It was supported in Germany by DFG. In Italy we thank MURST and the CINECA Computing Center. Primary production of simulations was done on a Cray-2 and Convex C3 at the National center for Supercomputing Applications, Urbana, IL. We thank in advance the Aspen Center for Physics, which has graciously agreed to sponsor a workshop on this and closely related topics in June 1994.

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FIGURE CAPTIONS

Figure 1 A plot of the crosscorrelation S of each of the various approximate solutions with the n body simulation, both being smoothed by the same (variable) size Gaussian window, against σ , the *rms* density fluctuation in the smoothed n -body simulation. Results are shown for spectral indices $n = +1$ and $n = -1$ at the moment when $k_{n\ell} = 8k_f$, where k_f is the fundamental mode of the box. In order of increasing accuracy, linear theory is the short dashed line, the frozen flow approximation is the dottled line, the adhesion approximation is the long dashed line, and the truncated Zel'dovich approximation (1TZA) is the solid line.

Figure 2 (a) A greyscale plot of a thin slice of the $n = +1$ n body simulation at the moment when $k_{n\ell} = 8k_f$. (b) A corresponding slice of the 1TZA approximation to the same. 2TZA does not look much different.

Figure 3 (a) As in Figure 2(a) but for $n = -1$. (b) As in Figure 2(b), but for $n = -1$.

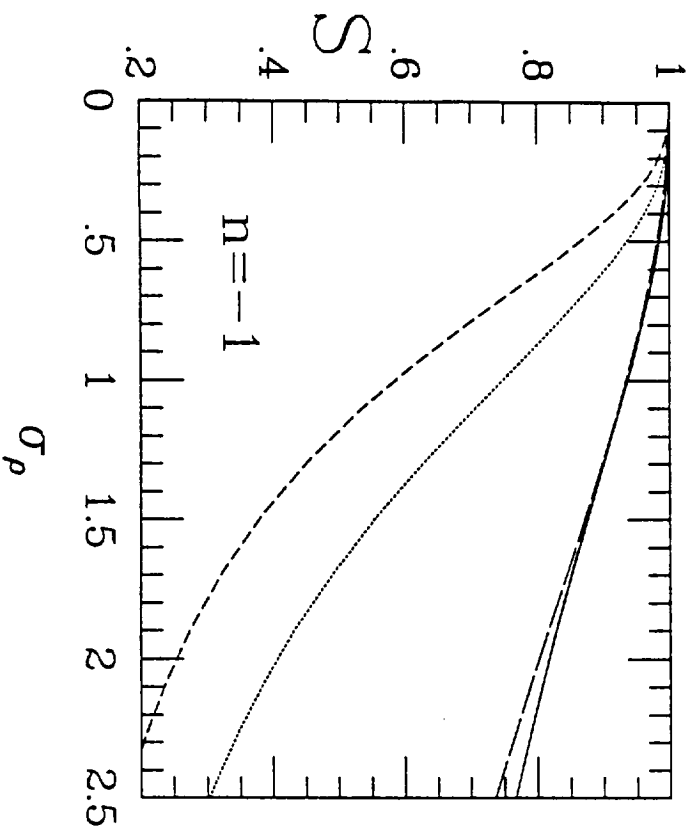
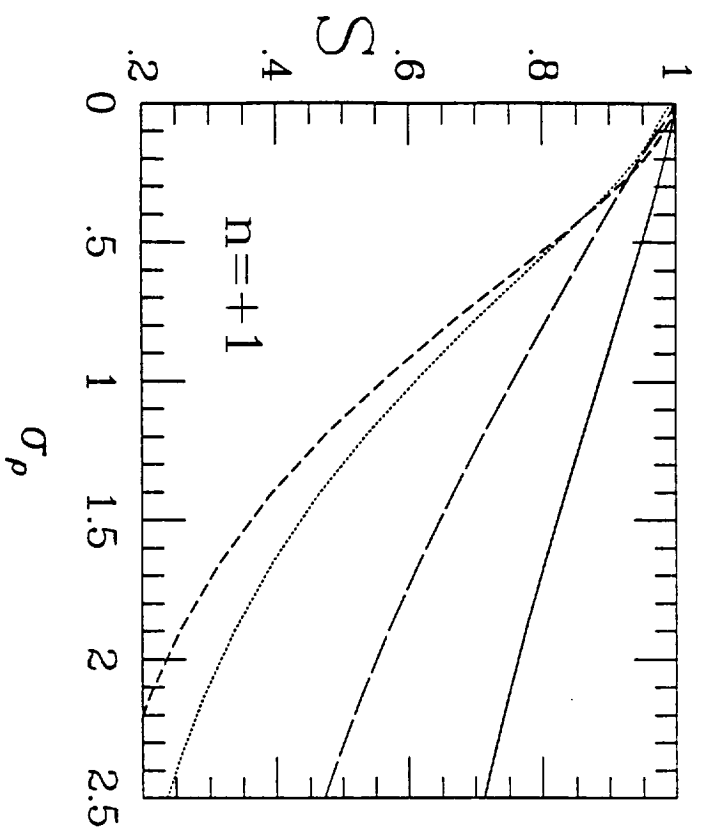


Figure 1

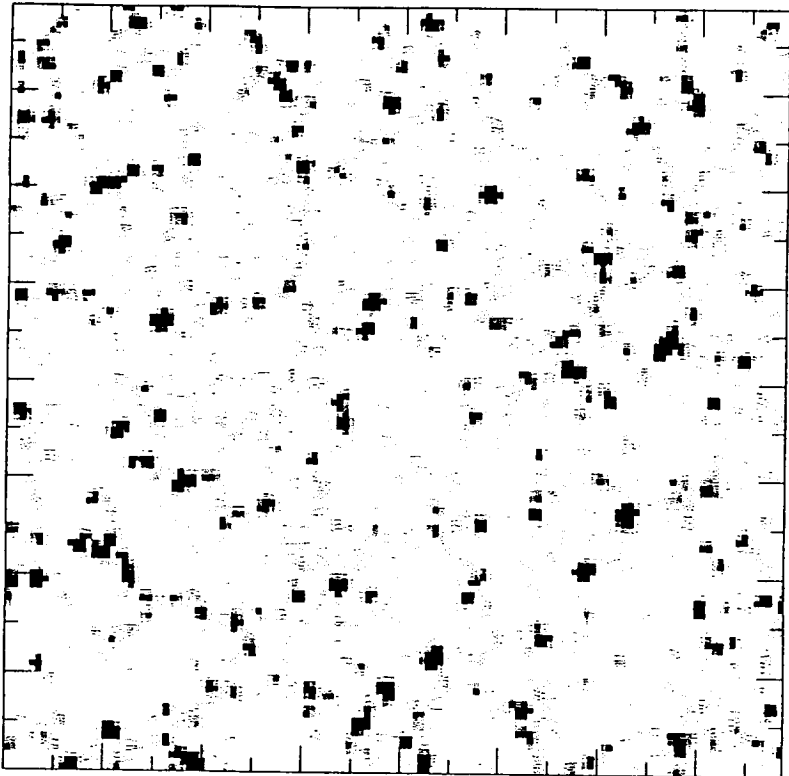


Fig. 2a

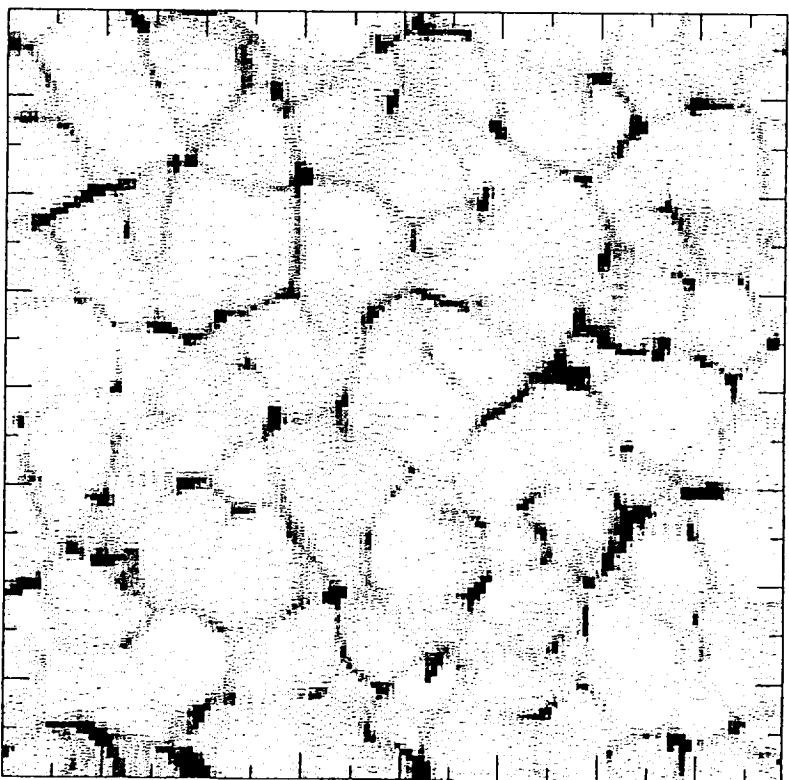


Fig. 2b

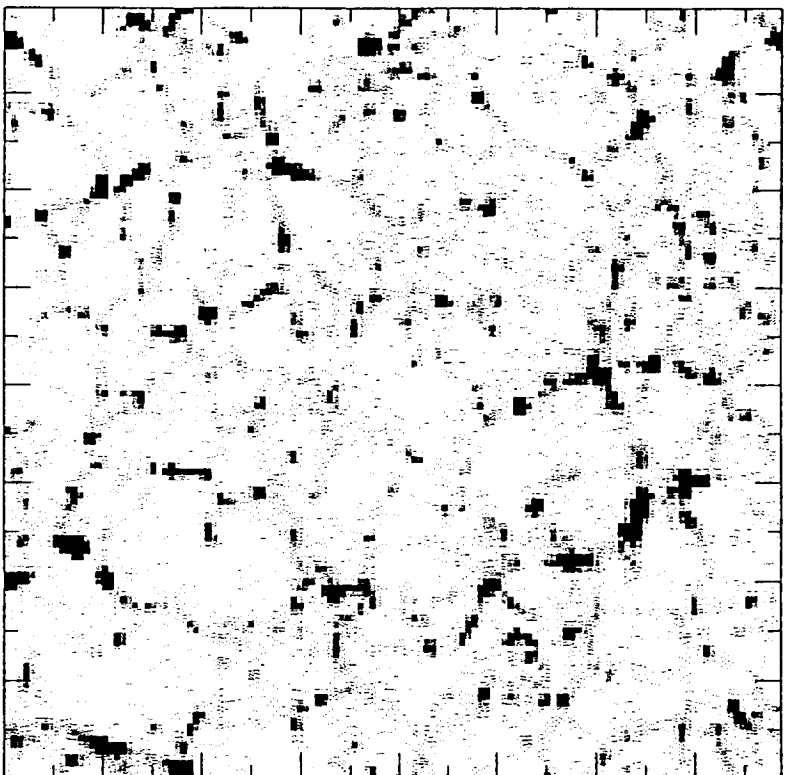


Fig 3a

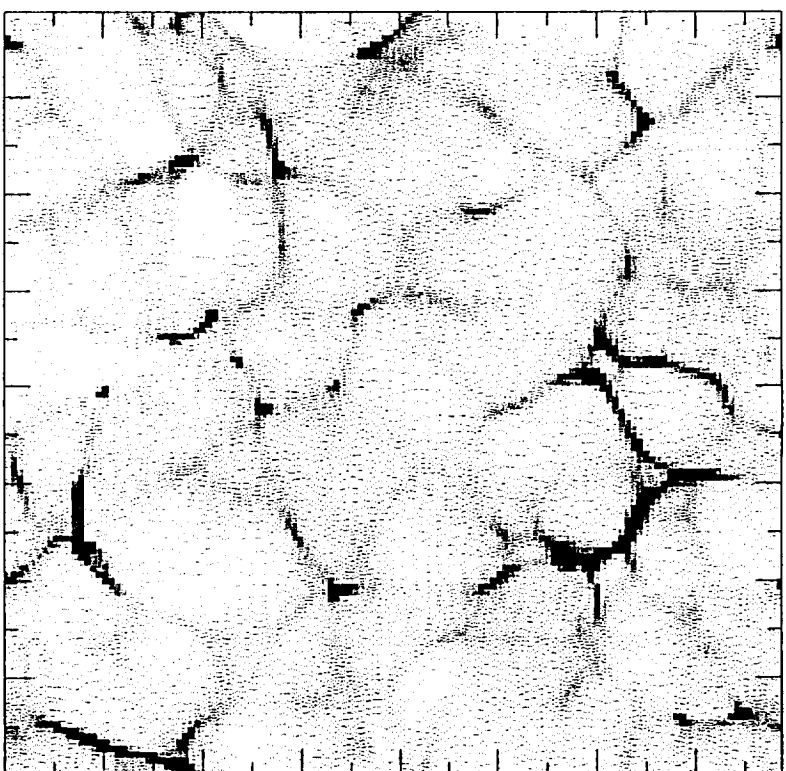


Fig 3b